On Financial Deepening and Long-Run Growth

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Abstract

I develop a stylized model of endogenous growth in which the level of financial depth influences an economy's long-run growth. Financial depth is defined within the model as the ease with which investors can issue equity in the market on new units of capital. I assume that agents differ in the cost of undertaking investment projects and that there is a fixed distribution of such costs across the population. I theoretically identify channels through which financial depth influences growth, both positively and negatively. When considering a specific distribution of costs, I show that the net effect of financial depth on growth is non-monotonic. It depends on the shape of the distribution, as well as the level or stage of financial depth. The results of this paper help to rationalize some findings in the recent empirical literature on the non-monotonic effect of financial depth on long-run growth. The model is even capable of obtaining a *negative* effect of excessive financial depth on growth, a result that is also found in the empirical literature.

Keywords: endogenous growth, financial depth, heterogenous agents.

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1 Introduction

Does the development of financial markets influence long-run growth? This question has been examined at both the theoretical and empirical level; see, among many other contributions, King and Levine (1993), Levine and Zervos (1998), Demirguc-Kunt and Levine (2008), Greenwood and Jovanovic (1990), Rioja and Valev (2004a), Rioja and Valev (2004b), and Kiyotaki and Moore (2005a). In general, the literature has identified theoretical considerations and evidence that financial depth has mixed effects on growth. I contribute to the theoretical discussion by providing a stylized model of heterogenous agents, in which credit, in the form of equity financing, flows among agents who differ in their ability to undertake investment projects. These financial flows face an exogenous impediment, which is interpreted as the degree of financial depth. Financial frictions of this sort have been proposed by Kiyotaki and Moore (2005b) and Kiyotaki and Moore (2012). I depart from their analysis in many respects, but more fundamentally, I allow for spillovers in the production function of the economy, and in the spirit of seminal papers such as Frankel (1962) and Romer (1986), I introduce "learning by doing" in a simple fashion by postulating that when firms rent capital and use it in production, they immediately enhance productivity in the economy, an aggregate spillover. The economy may grow forever, and the model provides a closed-form solution for the economy's long-run growth. I then examine what this simple model suggests with respect to the relationship between the depth of financial markets and long-run growth.

To analyze the extent to which financial deepening may influence growth, it is desirable to have heterogeneity that gives rise to credit in equilibrium. In much of the related literature, heterogeneity is introduced by assuming that the probability of finding an investment opportunity is exogenously given; see Kiyotaki and Moore (2012), Nezafat and Slavick (2015), Shi (2015) and Jinnai and Guerron-Quintana (2015). I depart from this assumption by endogenizing the decision of whether to undertake investment projects. The feature I introduce is to assume that individuals draw, in each period, an efficiency level or a cost of transforming the consumption good into capital. In certain periods, some agents may be very efficient at transforming the consumption good into capital, while in other periods, they can only do so at a high cost. This idiosyncratic productivity is governed by a given probability distribution. With this feature, the model provides a cutoff value for how efficient an individual needs to be to undertake investment projects. When an individual draws a cost below the cutoff value, he will undertake the project, becoming an investor. Investors may issue equity on the capital they create in the market, but they are exogenously constrained to hold at least some of the new capital as own equity, and this measures the degree of financial depth. Relatively inefficient agents, those whose cost draws are above the threshold, will partially finance investment by purchasing equity issued by investors.

The cutoff value mentioned above turns out to be the price of equity. Intuitively, if the cost of a unit of new capital is below the price of equity that can be issued on that unit, then it pays to undertake investment. I demonstrate that the equity price falls when the financial market becomes deeper. In simple terms, financial deepening makes equity less scarce, and thus, its market valuation decreases. With this result in place, it is possible to decompose the effect of deeper financial markets on growth into three components. Two of them have an unambiguously *negative* effect on growth.

The first effect is labeled the *wealth* effect. Growth is supported by investors, who are relatively efficient in transforming the consumption good into capital. They use available resources to do so. One of their resources is the capital accumulated from previous periods, the value of which declines with the drop in the asset price. This wealth effect produces a decrease in the desired amount of investment, which translates into less growth.

The second effect concerns the *extensive* margin. The lower the asset price is, the larger the fraction of agents who are relatively inefficient in creating capital relative to the cutoff value, and fewer individuals engage in investment. Having fewer investors entails less new capital production, which translates into lower growth.

The third effect is related to the *intensive* margin. Investors in the model face an idiosyncratic down-payment, or *effective* cost of investment. When creating capital, investors do not bear the entirety of this cost but sell equity on new units of capital created up to the financial constraint of the economy.

When this constraint is relaxed, meaning a deeper financial market, the downpayment required tends to decrease, but since the asset price under which equity is issued in the market also decreases, the net effect on the *effective* cost is ambiguous. If the equilibrium asset price is *inelastic* with respect to financial market depth, then the downpayment decreases, which boosts investment. Thus the third effect may dominate the other two negative effects and increase growth.

Determining how sensitive the equilibrium price of the asset is to greater financial depth and its impact through the three effects mentioned above is key to assessing the net effect on growth. To make progress on this issue, I consider a specific distribution of the cost of investment, the Weibull distribution. It turns out that the shape of the distribution influences the sensitivity of the equilibrium price of equity to financial deepening. I find that the model delivers a non-monotonic relationship between financial deepening and growth. In particular, under certain parameterizations of the Weibull distribution, is possible to find that when departing from low levels of financial depth, financial deepening increases growth at diminishing rates, and eventually, a threshold is reached after which financial deepening decreases growth.

Related Literature

The empirical literature on the relationship between financial deepening and growth is vast, while the purely theoretical literature is somewhat limited. Greenwood and Jovanovic (1990) present a model in which financial intermediation and growth are endogenously determined and positively correlated. Morales (2003), using a setup that emphasizes moral hazard, shows that there exists a negative relationship between the financing of innovation and the process of capital accumulation. Another related contribution is Giordani (2015), who develops a matching model in general equilibrium in which financiers and entrepreneurs match to create an innovation, which yields higher growth. Efficiency in the matching process is governed by an aggregate "matching function" that accommodates a thick market externality. In his model, actual financial assets are absent, and real resources flow among agents who match according to the matching function. The author explores what type of policy may induce optimality in that setup. Kiyotaki and Moore (2005a) follow an approach that is similar to the model developed in the present paper. However, there are important differences. First, they do not develop a model of endogenous growth but simply examine the impact of financial deepening on capital accumulation. Furthermore, they do not consider an endogenous determination of investors and lenders, and they find that financial deepening unambiguously increases capital and output.

In the empirical literature, the initial studies tend to find a positive relationship between financial deepening and growth, while later studies cast some doubt on this finding. King and Levine (1993) is a seminal empirical contribution. They present cross-country evidence in support of Schumpeter's view that the financial system can promote economic growth. Rajan and Zingales (1998) show that a more developed financial market positively impacts industrial sectors in need of external finance and hence fosters growth. Rioja and Valev (2004b) show that financial development has a differential effect on the sources of growth in developed and developing economies, and Rioja and Valev (2004a) show that there is a non-monotonic relationship between financial development and growth. Levine (2005) concludes that while there is evidence that financial development matters for growth, this is "subject to ample qualifications and countervailing views". Ang and McKibbin (2007) even find evidence of "reverse causality" in the case of Malaysia as the country underwent a financial liberalization process. They find that output growth leads to greater financial depth. This view of the causal relationship has also a long tradition initiated by Robinson J (1952). In the case of China, Liang and Teng (2006) show that the causality is unidirectional, running from economic growth to financial development. Brezigar Mastena et al. (2008) analyze the case of Europe and find significant non-linear effects, with less-developed European countries gaining more from financial development. One important study is Bekaert et al. (2005), who focus on instances of equity market liberalization and find a positive and significant causal effect of financial liberalization on growth. Demirguc-Kunt and Levine (2008) argue that while theoretical models are ambiguous with respect to the relationship between financial development and growth, the empirical literature is more conclusive and asserts that the relationship is positive. Ben Gamra (2009) studies six major emerging East Asian countries and find that the effect of financial liberalization on growth depends on the nature and intensity of such liberalization. Full liberalization of the financial sector is associated with slower growth outcomes, while more moderate partial liberalization is associated with more positive outcomes. Rousseau and Wachtel (2009) show that the sample under consideration is important when assessing the relationship between finance and growth. Examining recent data (post 1990), they find a relatively dampened effect of financial deepening on growth. Beck et al. (2012) show that for a set of developed and developing countries, enterprise credit is associated with economic growth whereas household credit is not. Hook Law and Singh (2014) study 87 developed and developing countries. They find a threshold in the finance-growth relationship; the level of financial development benefits growth only up to a certain threshold, beyond which further financial development tends to adversely affect growth. In the same vein, Arcand et al. (2015) demonstrate that there can be "too much finance", a given threshold above which finance begins to have a negative effect on output growth.

The remainder of the paper is organized as follows. Section 2 presents the model, section 3 solves for the model's economic equilibrium, section 4 analytically examines the effect of financial deepening on growth, section 5 examines the implications of a specific distribution of investment costs, and section 6 concludes the paper.

2 The Model

2.1 Environment

The economy is populated by a measure one of infinitely lived individuals, who seek to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t, \quad 0 < \beta < 1. \tag{2.1}$$

Linear utility has been exploited successfully in the context of heterogeneous agents (Taub (1988) and Taub (1994)) and in the context of heterogenous agents with financial frictions (Salas (2017)).[1] The

[1] Despite being a stringent assumption, linear utility has the virtue, as will be seen shortly, of allowing for closed-form solutions for the policy functions and, more interestingly, a closed-form solution for the entire distribution of individuals by assets.

expectation operator $\mathbb{E}_0$ refers to an uninsurable idiosyncratic risk. All agents in each period are able to create capital, but they differ in the cost of doing so. Specifically, when x units of the consumption good are allocated to capital creation, an individual's next-period capital stock is[2,3]

$$k_{t+1} = (1-\delta)k_t + x_t, \tag{2.2a}$$

where δ is the deprecation rate. However, transforming x_t units of the consumption good into capital has an effective cost of $z_t x_t$, and $z_t \in \mathcal{Z} \equiv [1, \bar{z}]$ is the idiosyncratic cost, which is a draw from a CDF $F(z)$. The approach of modeling heterogeneity by assuming idiosyncratic shocks to the cost of investment is also used by Buera and Moll (2015). In the present context, these shocks are better understood to capture how efficient investment ideas are in terms of their cost of implementation. As a normalization, the minimum possible cost is unity; at this value, consumption goods can be transformed one-to-one into capital goods. Upon observation of z, each agent has to decide whether and how much to invest and how much to consume. There is a single financial asset in this economy: claims on capital n, which all agents can use to effect intertemporal consumption. Thus, agents can decide how much of their claims on capital to trade in the market, and if an agent creates capital, he can also decide how much equity to issue in the market. Heterogeneity, of course, opens the possibility for credit flows in equilibrium. Any capital created in the market will be rented along with labor to CRS firms, the optimization problem of which will be introduced below. Let q_t be the price of claims on capital, r_t be the rental rate on capital and w_t be the wage rate. The individual budget constraint is

$$c_t + z_t x_t + q_t[n_{t+1} - (1-\delta)n_t] = w_t + r_t n_t + q_t x_t, \tag{2.2b}$$

[2]Note that to simplify notation, I avoid using subindexes to denote an individual's quantities, such as $c_{i,t}$ in (2.1) or $x_{i,t}$, $k_{i,t}$ in (2.2a). Instead, I use lower-case letters to denote individual variables and capital letters to denote aggregates. I use a subindex $t+1$ to denote next-period values and sometimes a prime.

[3]An alternative modelling assumption would be to make those firms responsible for the production of the consumption good also undertake the investment. This would complicate the analysis without providing a clear gain in addressing the question at hand because the spillover from the use of capital will manifest among the same agents who face financial frictions. I chose to follow the original contribution of Romer (1986) and assume that consumers also undertake investment decisions.

and the financial constraint that agents face is

$$n_{t+1} \geq (1-\theta)x_t, \quad \theta \in (0,1). \tag{2.2c}$$

The financial structure displayed in (2.2b) and (2.2c) is very similar to that proposed by Kiyotaki and Moore (2005b) and Kiyotaki and Moore (2012). Here, I provide a brief explanation of these equations. In Appendix A, I explain the financial market structure leading to these feasibility constraints in greater detail. The first two terms on the RHS of (2.2b) simply represent income from renting the factors of production. The first two terms on the LHS are the expenditures on consumption and investment, respectively, while the last term is the change in claims or equity over capital. Both this last term on the LHS of (2.2b) and the last term on the RHS of the same equation cannot be read independent of (2.2c). This last constraint states that claims on capital have to be at least $(1-\theta)$ of investment. That is, an agent cannot issue claims on all new units of capital created. In this paper, a deeper financial market means a higher θ. In this case, credit will flow easily, and equity can be issued on a large fraction of the new capital created. In a limit where financial constraints are tightest, $\theta \to 0$, an agent must claim all new units of capital that he decides to create.[4,5]

Let $\mathcal{V}_t(n,z)$ be the value function for an agent with state n and status z. The Bellman equation for an agent with states (n,z) is

$$\mathcal{V}_t(n,z) = \max_{c_t \geq 0, x_t \geq 0} \left[c_t + \beta \int_{\mathcal{Z}} \mathcal{V}_{t+1}(n',z') dF(z') \right], \tag{2.3}$$

subject to (2.2a), (2.2b) and (2.2c). Note also the non-negativity constraint on both investment and consumption.

[4] Of course, he can always hold more claims than required by the financial constraint. That is, the constraint may be satisfied with strict inequality. Thus, an agent not only claims all units of capital he has created, but he may also purchase more equity in the market beyond that point.

[5] Kiyotaki and Moore (2005b), Kiyotaki and Moore (2012), Shi (2015) and Jinnai and Guerron-Quintana (2015) all also assume the existence of another friction (parameterized by ϕ), which prevent agents from selling desired equity on *existing* units of capital. I disregard this constraint by assuming that $\phi = 1$. This assumption is also used by Nezafat and Slavick (2015). The reason for doing so is that the cited papers that use the assumption focus on *liquidity shocks*, namely, fluctuations in the parameter ϕ. In contrast, I am more interested in long-run financial frictions, which are more akin to the financial deepening captured by θ.

There is a measure one of firms, indexed by j. I assume the existence of a spillover in their production function. CRS firms rent capital, produced by current entrepreneurs, and labor services, provided by all agents, in each period to maximize $[Y_{jt} - r_t K_{jt} - w_t L_{jt}]$, where

$$Y_{jt} = B_t K_{jt}^{\alpha} L_{jt}^{1-\alpha}, \quad B_t = A K_t^{1-\alpha}, \quad K_t = \int_0^1 K_{jt} dj. \tag{2.4}$$

This production function produces long-run growth. Firms renting and using capital concurrently increase the productivity of all other firms.

The distribution of individuals with respect to assets is denoted $\Psi_t(n)$. If a solution to (2.3) exists, it will deliver policy functions for consumption $c_t(n,z)$, next-period equity $g_{t+1}(n,z)$ and investment $h_t(n,z)$. The law of motion for the distribution of individuals by assets is

$$\Psi_{t+1}(n') = \int_{\mathcal{Z}} \int_{\mathcal{N}(n')} d\Psi_t(n) dF(z), \tag{2.5}$$

where $\mathcal{N}(n') = [n : n' \geq (1-\theta)h_t(n,z), g_{t+1}(n,z) \leq n', \ z \in \mathcal{Z}]$.

2.2 Definition of equilibrium

Definition *A competitive equilibrium is a sequence of prices* $\{q_t, r_t, w_t\}_{t=0}^{\infty}$ *and distributions* $\{\Psi_t(n)\}_{t=0}^{\infty}$ *such that, given an initial distribution* $\Psi_0(n)$*,*

1. $c_t(n,z)$*,* $g_{t+1}(n,z)$ *and* $h_t(n,z)$ *maximize an individual's utility subject to constraints.*
2. *Claims on the capital market clear:*

$$\int_{\mathcal{Z}} \int_{\mathcal{N}(n')} n d\Psi_t(n) dF(z_t) = K_t. \tag{2.6a}$$

3. Investment demand equals savings:

$$\int_{\mathcal{Z}}\int_{\mathcal{N}(n')} g_{t+1}(n,z)d\Psi_t(n)dF(z)-(1-\delta)\int_{\mathcal{Z}}\int_{\mathcal{N}(n')} nd\Psi_t(n)dF(z) = \int_{\mathcal{Z}}\int_{\mathcal{N}(n')} h_t(n,z)d\Psi_t(n)dF(z) \tag{2.6b}$$

3 Solving the Model

Let me begin with the following observation. Consider constraint (2.2b), and note that when an agent makes his decision of whether and how much to invest, he will compare his draw z_t with q_t, as when $z_t < q_t$, the cost of investment is lower than the income obtained by issuing equity on new units of capital. Such agents will invest as much as possible subject to constraint (2.2c); in fact, they will reach this constraint. On the contrary, when $z_t > q_t$, the cost of investment is higher than its benefit, and investment will be zero. q_t then is a natural cutoff value for z_t, which is useful for making an investment decision. Those with z_t below q_t will invest; I label them "investors". Those who face z_t above q_t will not invest, and these agents will be called "lenders". Agents may change status from period to period, as a new draw is obtained in each period. Note that the fractions of lenders and investors will be determined endogenously, as q_t is determined by the market equilibrium.[6]

Since those agents with $z_t \leq q_t$ will invest as much as possible subject to constraint (2.2c), it will bind.[7] Substituting out n_{t+1} from (2.2c) at equality and using (2.2b),

$$c_t + (z_t - q_t\theta)x_t = w_t + r_t n_t + q_t(1-\delta)n_t. \tag{3.1}$$

Equation (3.1) reveals that the cost of investment is not z_t but the *effective cost* $z_t - q_t\theta$, which is

[6] This stands in contrast to several studies that introduce heterogeneity by assuming the arrival of investment opportunities; see, for example, Kiyotaki and Moore (2012), Nezafat and Slavick (2015) and Shi (2015). These papers typically assume that agents face an exogenous probability π of having an investment opportunity. By the law of large numbers, this also is the fraction of agents who are investors in the economy. Here, the fraction of investors, in equilibrium with price q_t, is given by $F(q_t)$.

[7] Specifically, this is true for $z_t < q_t$, as in the case in which $z_t = q_t$, agents are completely indifferent between investing and not. Since this situation has measure zero, for general $F(z)$, this assumption is innocuous.

decreasing in the fraction of capital sold by issuing equity $q_t\theta$.

An alternative depiction of the constraint for agents who face $z_t \leq q_t$ is obtained by substituting out investment from (2.2b) with investment from (2.2c) at equality:[8]

$$c_t + p_t n_{t+1} = w_t + r_t n_t + q_t(1-\delta)n_t, \quad n_{t+1} \geq 0, \tag{3.2a}$$

where

$$p_t \equiv \frac{z_t - q_t\theta}{1-\theta}. \tag{3.2b}$$

p_t is defined as the effective price of equity for investors, which is idiosyncratic. Since the investor claims only $1-\theta$ of the new units of capital he creates, the cost of one *unit* of equity is the *effective* cost, or downpayment $z_t - q_t\theta$ divided by $1-\theta$.[9] Note that for the marginal investor, $z_t = q_t$, $p_t = q_t$, while the other investors will face $p_t < q_t$, and this means that investors have an advantage respect to lenders when $q_t > 1$. The opportunity cost of current consumption is higher for them than for lenders in this case. As we will see, the only equilibrium involves precisely $q_t > 1$. For agents facing $z_t > q_t$, the "lenders", the feasibility set is

$$c_t + q_t n_{t+1} = w_t + r_t n_t + q_t(1-\delta)n_t, \quad n_{t+1} \geq 0, \tag{3.2c}$$

since $x_t = 0$ for them. To solve the model, I will focus on a stationary case in which the economy is growing at a constant rate.[10] Let me begin with the simple problem for the firms. Firms equate the private marginal products of both capital and labor to their rental rates:[11]

$$r_t = \alpha B_t K_t^{\alpha-1} = A\alpha \equiv r, \quad w_t = (1-\alpha)B_t K_t^{\alpha} = A(1-\alpha)K_t. \tag{3.3}$$

[8]The inequality constraint in (3.2a) is just $x_t \geq 0$, written in terms of equity by using (2.2c) at equality.

[9]Because $z \in \mathcal{Z} \equiv [1, \bar{z}]$, it is necessary that $1 > q_t\theta$ for the investors' problem to be well defined. That is, even for the most efficient agent, the price of equity must be positive.

[10]It can be shown that, in contrast to the initial AK models, the present model has transitional dynamics. To address the question at hand, however, I chose to simplify the analysis by focusing on the steady state.

[11]Under this conditions, I have already imposed the equilibrium condition that all agents inelastically supply labor, and thus, $L_t = 1$ at all times. When making their hiring decisions, firms do not consider the aggregate effect on productivity due to capital accumulation in A_t, this is Romer (1986)'s spillover.

In any equilibrium, the rental rate of capital will be constant and the wage rate will grow with capital. The economy's constant (gross) growth rate is denoted γ.

The economy can be summarized by two equations that are presented in Proposition 1.

Proposition 1. *If an equilibrium exists, the economy can be described by the following two equations:*

$$\frac{1}{\beta} = \frac{[r+q(1-\delta)]}{q}\int_1^q \frac{q}{p}dF(z) + \frac{[r+q(1-\delta)]}{q}[1-F(q)] \tag{3.4a}$$

$$\gamma = [A+q(1-\delta)]\int_1^q \frac{1}{z-\theta q}dF(z) + 1 - \delta \tag{3.4b}$$

Proof. To prove Proposition 1, I will make use of the lemma below.[12]

Lemma 1. *The value function* (2.3) *exists and is given by*[13]

$$\mathcal{V}_t(n,z) = \mathcal{C}_t(z) + \mathcal{D}_t(z)n, \tag{3.5a}$$

where

$$(\mathcal{C}_t(z), \mathcal{D}_t(z)) = \begin{cases} \left(\left\{\gamma\beta\left[\int_1^q \frac{q}{p}dF + 1 - F(q)\right] + (1-\gamma\beta)\frac{q}{p}\right\}\frac{(1-\alpha)AK_t}{1-\gamma\beta}, \quad \frac{q}{p}[r+q(1-\delta)]\right) & \text{if } z \leq q \\ \left(\left\{\gamma\beta\left[\int_1^q \frac{q}{p}dF + 1 - F(q)\right] + 1 - \gamma\beta\right\}\frac{(1-\alpha)AK_t}{1-\gamma\beta}, \quad [r+q(1-\delta)]\right) & \text{if } z > q \end{cases} \tag{3.5b}$$

The associated policy correspondences are

$$g_{t+1}(n,z) = \begin{cases} \frac{w_t+[r+q(1-\delta)]n_t}{p}, & z \leq q \\ \in \left[0, \frac{w_t+[r+q(1-\delta)]n_t}{p}\right], & z > q \end{cases} \quad c_t(n,z) = \begin{cases} 0, & z \leq q \\ \in [0, w_t + [r+q(1-\delta)]n_t], & z > q \end{cases} \tag{3.6a}$$

$$h_t(n,z) = \begin{cases} \frac{w_t+[r+q(1-\delta)n_t]}{z-\theta q}, & z \leq q \\ 0, & z > q \end{cases} \tag{3.6b}$$

Proof. See Appendix B. □

[12] The proofs of Lemma 1 and the rest of the propositions in this paper are presented in Appendix B.

[13] The closed-from solution for the value function (2.3), expressed in (3.5a), reveals that the coefficients in the value function depend on z for investors but not for lenders. This makes sense since the downpayment for investment depends on z for the former but is zero for the later, independent of their obtained draw of z.

Corollary 1. *Asset pricing relationships. If an equilibrium exists, the following relationships must be satisfied:*

$$p < q = \beta \mathbb{E} \mathcal{D}_{t+1}(z') \tag{3.7}$$

Lemma 1 shows that $\mathcal{D}_t$ does not depend on time, and it provides analytical expressions. Therefore, the equality in Corollary 1 coupled with $\mathcal{D}_t(z)$ in (3.5b) yields equation (3.4a) in Proposition 1.

To derive the rate of growth of the economy (3.4b), let me focus on the policy function for investment, equation (3.6b). Investors use all resources to create capital and accumulate claims on capital, while investment is zero for lenders. Therefore,

$$X_t \equiv \int_{\mathcal{Z}} \int_{\mathcal{N}(n')} h_t(n, z) d\Psi(n) dF(z) = \{w_t + [r + q(1 - \delta)]K_t\} \int_1^q \frac{1}{z - \theta q} dF(z), \tag{3.8}$$

where X_t is defined as aggregate investment. By aggregating (2.2a), I also obtain

$$K_{t+1} = (1 - \delta)K_t + X_t \tag{3.9}$$

Dividing (3.8) by K_t and using (3.9) yields the rate of growth equation (3.4b). □

Equation (3.4a) is a balancing of the discount rate and the expected return for lenders. If an equilibrium exists, they must be indifferent between devoting all their resources to consumption or to purchasing equity, a feature implied by linear utility. Because investors are selling equity in the market, equity prices must be such that expected returns for lenders exactly match the discount rate; otherwise, there would always be zero consumption for all individuals, or there would be no demand for equity. Indifference for lenders is reflected in equation (3.6a). This equation and (3.6b) also reflect the "corner" solution for investors who consume nothing and divide their income between capital expenditures and self-claimed equity.[14]

[14]Hence, investors partially save through self-claimed equity. They are forced to do so because they can only sell equity

The expected return in (3.4a) is composed of two parts. The first term is the expected return conditional on being an investor. The amount of consumption goods that equity provides is $r+q(1-\delta)$, which when divided by the cost of the unit of equity acquired q, yields returns in consumption terms. These goods, however, are not consumed by investors but valued at q/p, which is higher than one when $q > 1$.[15] Since p is a function of z, we need to integrate over all possible values of $z \leq q$. The second term is the expected return conditional on being a lender. In such a case, the agent will not invest, and the unit of equity carried into the future at cost q will provide him with $r + q(1 - \delta)$ in consumption goods.

Equation (3.4b) has an intuitive explanation. Capital created by investors is rented to the CRS firms, which produce the spillover responsible for endogenous growth. Investors use their resources $A+q(1-\delta)$ to increase capital. The investment cost is $z-\theta q$. The lower this cost is, the more capital can be created, which sustains more growth. The transformation of the consumption good into capital is idiosyncratic. For the most efficient agent with $z = 1$, the cost is $1 - q\theta$, and then it is required that $q < 1/\theta$, which we will see holds in equilibrium. For the rest of the agents with $z > 1$, the cost is higher and always positive.

3.0.1 A brief detour: A homogenous agent result

Let us consider for a moment the model's equilibrium when heterogeneity is eliminated. Assume that there is a single mass of individuals with efficiency level $z = 1$.[16] In such a situation, $q > 1$ cannot be an equilibrium because all agents will attempt to invest as much as possible, and no one will be willing to lend to them. $q < 1$ cannot be an equilibrium either, as there would never be any capital creation. The only possible equilibrium is when $q = 1$, which implies $q = p = 1$. In such a situation, equation

up to a fraction θ of the new units of capital they create.

[15] Since p is the effective price of equity for investors, $[r + q(1-\delta)]/p$ are the units of goods "transformed" into equity, which are valued at price q. Furthermore, when $q > 1$, $q/p > 1$, which demonstrates the advantage that investors have from being relatively efficient in the production of new capital.

[16] A more formal route that yields the same result would be obtained by assuming that $dF(z)$ is the Dirac delta function at $z = 1$.

(3.4a) translates to

$$\frac{1}{\beta} = (r + 1 - \delta) = (A\alpha + 1 - \delta), \tag{3.10}$$

which is the usual steady-state result that equates the rate of time preference with the return on the asset. If such an equality is to be satisfied, the value of A is pinned down as $A = [1-(1-\delta)\beta]/(\alpha\beta)$. In such a case, all agents would be indifferent between consumption and saving, which should be the case in this linear utility case in the steady state. Such a value for A will be imposed in the model under the understanding that F is arbitrary, and the case in which the entire mass of agents is arbitrarily close to $z = 1$ should be admitted by the model.[17]

3.1 Existence

If an equilibrium exists, the values of q and γ should satisfy equation (3.4). A value of q that satisfies equation (3.4a) can be found independent of (3.4b). Hence, demonstrating the existence of an equilibrium only requires showing that such a value of q that satisfies (3.4a) can be found. I turn now to this issue.[18]

Proposition 2. *Existence and uniqueness of equilibrium.*[19] *Under the condition*

$$\theta\left[1 - F\left(\frac{1}{\theta}\right)\right] + (1-\theta)\int_1^{\frac{1}{\theta}} \frac{1}{z-1} dF(z) > \frac{\theta}{\theta + \beta(1-\delta)(1-\theta)}, \tag{3.11}$$

there exists a unique $q \in [1, 1/\theta]$ *such that equation* (3.4a) *is satisfied.*

Proof. See Appendix B. □

[17] The discussion of the homogenous agent result serves only to consider a proper value for A. The absence of diminishing returns in the production function leaves capital accumulation, and hence the economy's growth rate, undetermined.

[18] Of course, with the equilibrium value of q, γ can be found with (3.4b), which could turn out to be higher or lower than one, that is, a growing or a shrinking economy. This will be addressed later in the paper.

[19] This proposition is valid, even if we consider a lower value for A than that shown in (3.10), as noted in the proof in Appendix B. Numerical analysis for the case of higher values of A reveals that two equilibria may arise, one with high growth and another with low growth. By focusing on the high-growth equilibrium, everything in the paper remains valid. The numerical analysis for this case is available upon request.

Condition (3.11) is a regularity condition. This condition is a complicated function of the parameters of the model, in particular of θ and the function F. In the next section, this condition will be verified when I consider a specific function F.

The next proposition shows that financial deepening decreases the price of the asset.

Proposition 3. *Financial deepening decreases the price of the asset:*

$$\frac{dq}{d\theta} < 0. \tag{3.12}$$

Proof. See Appendix B. □

When θ increases, equity, which is relatively more plentiful than before, loses value. Another way to examine this result is to determine the effects of a higher θ on the net investment demanded by investors and the savings supplied by lenders. These quantities must be in balance in equilibrium, as expressed in equation (2.6b).

Investor's behavior under financial deepening

Individual net investment demand is defined as $nx_t(n,z) = h_t(n,z) - [g_{t+1}(n,z) - (1-\delta)n]$ and can be computed from (3.6a) and (3.6b) as follows:

$$NX_t \equiv \int_1^q \int_{\mathcal{N}(n')} nx_t(n,z)d\Psi(n)dF(z) = \{w_t + [r + q(1-\delta)]K_t\} \int_1^q \frac{\theta}{z-\theta q}dF(z) + (1-\delta)K_tF(q) \tag{3.13}$$

It is clear that a higher θ, given q, increases aggregate net investment.

Lender's behavior under financial deepening

Lenders' behavior is guided by equation (3.4a), where they balance the discount rate against expected returns. Let me re-write equation (3.4a) as follows:

$$\frac{1}{\beta} = \mathcal{R}\left[\int_1^q \psi(z)dF(z) + \int_q^{\bar{z}} 1 \cdot dF(z)\right], \tag{3.14a}$$

where $\psi(z) = q/p$, and

$$\mathcal{R} = \frac{r + q(1-\delta)}{q}. \tag{3.14b}$$

Note that expected returns can be computed as the multiplication of a "standard" return $\mathcal{R}$ and a return that accounts for the heterogeneity in how efficient are agents in transforming the consumption good into capital.[20] The function $\psi(z)$ measures how efficient investors are in performing such a transformation and is defined as the return for an investor conditional on a given efficiency z. It has the following properties:

$$\psi(1) > 1, \quad \psi(q) = 1, \quad \frac{\partial \psi}{\partial z} < 0, \quad \frac{\partial^2 \psi}{\partial z^2} > 0 \tag{3.14c}$$

The most efficient investor has the highest return $\psi(1)$, while the most inefficient agent is as inefficient as a lender and has return $\psi(q)$.

Under a higher θ, the returns $\psi(z)$ for any agent with efficiency $z \in [1, q)$ will increase; therefore, the RHS of (3.14a) would increase. This increases the supply of savings or lending. However, the increase in NX_t cannot be met with an equal increase in lending because given an increase in expected returns, lenders would consume nothing. Consumption would be zero for all agents at all times. Lenders therefore do not substantially increase lending, which produces a decline in q. Would a decline in q re-balance (3.14a)? Proposition 3 shows that it does. It is useful to explore the mechanisms of such a re-balancing.

A decrease in q produces the following effects: an increase in $\mathcal{R}$, a decrease in $\psi(z)$ for any z, and a "re-weighting" toward the low return conditional on being a lender. Because $\psi(z)$ is higher than unity, the term in parentheses in (3.14a) represents a sort of weighted average between high and low returns. The decrease in q places greater weight on the low return of unity. For an equilibrium to be

[20] As an unrelated but interesting point here is that it would not be proper to use $1 = \beta\mathcal{R}$ in calibrating β, which is a common practice in macro models.

reached under deeper financial markets, lenders must face a higher likelihood of remaining lenders than becoming investors. Concurrently, by the law of large numbers, the mass of lenders is increased enough to meet the higher investment demand from investors. This feature of equilibrium will be important when studying the effect of financial deepening on growth. Proposition 3 establishes that an economy with deeper financial markets will have a lower price of equity. This does not mean that *observed* returns on equity decrease in the depth of financial markets. In fact, the ex post *observed* return on equity in the model is given by equation (3.14b). Therefore, as θ increases, the return on equity also increases, which appears to be a widely accepted tenet in financial economics; see, for example, the elaborations of Mendoza et al. (2009).[21]

Having characterized the equilibrium asset price, I now turn to determining the effect of financial deepening on the growth rate of the economy.

4 The effect of financial deepening on growth

There is a direct effect of financial deepening, a higher θ, on growth. The investment cost is reduced, which increases capital and growth, as can be immediately seen in (3.4b). However, the truly important considerations are the general equilibrium effects, in particular the equilibrium response of q to a higher θ.

To determine the actual change in the growth rate of the economy when considering all general equilibrium effects, let me totally differentiate (3.4b):

$$\frac{d\gamma}{d\theta} = \underbrace{\frac{dq}{d\theta}(1-\delta)\int_1^q \frac{1}{z-\theta q}dF(z)}_{\textit{wealth}\text{ effect}} + [A + q(1-\delta)]\left[\underbrace{\int_1^q \frac{(1+\xi)q}{(z-\theta q)^2}dF(z)}_{\textit{intensive}\text{ margin}} + \underbrace{\frac{f(q)}{(1-\theta)q}\frac{dq}{d\theta}}_{\textit{extensive}\text{ margin}}\right] \tag{4.1}$$

[21]Some researchers, however, have challenged some established related results. For example Ritter (2005) challenges the notion that higher growth leads to higher equity returns.

It is then possible to determine three different effects of a deeper financial market on long-run growth.

The first effect is labeled the *wealth* effect. It is the change in the value of existing claims on capital that is used for capital creation. All investors face this effect. As q decreases, their wealth also decreases.

The second effect is labeled the *intensive* margin effect. It is the change in the idiosyncratic rate at which consumption goods are transformed into capital when financial deepening increases. Note that when the elasticity of q with respect to θ, ξ, is higher than one in absolute value, this term also produces a negative effect on long-run growth.

The third effect is labeled the *extensive margin* effect. It is the change in the number of individuals who engage in investment under financial deepening. When q decreases, more agents become relatively inefficient in transforming the consumption good into capital and instead choose to be lenders; this has a negative effect on growth.

It is clear from (4.1) that a necessary condition for growth to increase with financial deepening is that ξ be lower than one. To understand why growth decreases unambiguously given a higher θ if this condition does not hold, it is revealing to examine the numerator in the integral in equation (3.4b). Because an increase in θ entails a decrease in q, the decrease in q may offset the increase in θ, and in turn, the *effective* cost of investment may increase. Formally,

$$\frac{d(z-\theta q)}{d\theta} = -\left[q + \theta\frac{dq}{d\theta}\right] = -(1+\xi)q \tag{4.2}$$

Then, if the elasticity ξ is greater than one in absolute value, the *effective* cost of investment increases given a deeper financial market, for any level of efficiency z. Financial deepening can have negative effects on growth, even if this elasticity is lower than one in absolute value, if the wealth and extensive margin effects dominate the positive intensive margin effect. Unfortunately, no further *analytical* characterization can be made of these issues; hence, I resort to numerical examples in the next section of the paper. Before doing so, allow me to address some loose ends of the model.

4.0.1 Related elements of the model

Consumption

Output in the economy needs to be exhausted by consumption and investment; therefore, $Y_t \equiv AK_t = C_t + K_{t+1} + (1-\delta)K_t$. Dividing this equation by the stock of capital and using (3.9), consumption is given by

$$C_t = [A - \gamma - (1-\delta)]\, K_t \tag{4.3}$$

Financial deepening that may increase γ decreases aggregate consumption as a fraction of capital. Since utility is linear, we must be careful to check whether consumption is indeed positive.

The distribution of individuals by assets $\Psi_t(n)$

In many models of heterogenous agents that admit easy aggregation, there is no invariant distribution of assets by individuals; see, for example, the models in Lucas (1992), Angeletos (2007) and Buera and Moll (2015). One of the virtues of the analytic assumptions employed in this paper, mainly the linear utility assumption, is that this approach allows for the existence of the distribution of individuals by assets. This can be accomplished using the following innocuous assumption.

Assumption 1.

$$g_{t+1}(n, z) = \zeta K_{t+1}, \quad \zeta > 0, \; \textit{for any } z > q.$$

The aggregate equilibrium does not depend on this assumption. Absent a specification of how lenders resolve their indeterminacy between consumption and savings, the model is undetermined at the individual level. Assumption 1 simply states that all lenders behave equally, holding equity in proportion to aggregate capital.[22]

[22] All aggregate quantities in equation (3.4) are invariant to different assumptions on how the indeterminacy at the individual level is resolved. Different assumptions may affect individual welfare but do not change the results of the paper.

With Assumption 1, Proposition 4 establishes that $\Psi_t(n)$ exists and characterizes it.

Proposition 4. $\Psi_t(n)$ *satisfying* (2.5) *exists:*

$$\Psi_t(n) \equiv \Psi(n_{it}) = 1 - F(q)^i, \quad i = 1, 2, 3, ... \tag{4.4a}$$

with density

$$d\Psi(n_{it}) = 1 - F(q)^i - \left[1 - F(q)^{i-1}\right] = [1 - F(q)]\, F(q)^{i-1}, \quad i = 1, 2, 3, ... \tag{4.4b}$$

and support $\{n_{it}\}_{i=1}^{\infty}$*, defined by*

$$n_{it} = \left\{ \zeta \left(\frac{A\alpha + q(1-\delta)}{p\gamma} \right)^{i-1} + \frac{A(1-\alpha)}{p\gamma - A\alpha - q(1-\delta)} \left[1 - \left(\frac{A\alpha + q(1-\delta)}{p\gamma} \right)^{i-1} \right] \right\} K_t, \quad i = 1, 2, 3, ... \tag{4.4c}$$

Proof. See Appendix B. □

There exists a special distribution, whereby agents are distributed at discrete points over assets. We can see that financial deepening affects the distribution, both in the density itself and in the support. It is interesting that the entire support is increasing in proportion to capital if the economy is growing. I am not interested in issues of inequality within this model, not least because of its highly stylized nature. This proposition is only established to ensure the completeness of the model.

5 Financial deepening and growth: numerical examples

To assess the effects of financial deepening on growth, using equation (4.1), I resort to a numerical example. In principle, different distributions F could be considered for the exercise at hand. Given F, q determines two fractions of agents, investors and lenders. We know that, unambiguously, financial deepening *decreases* q, thereby decreasing the fraction of investors and increasing the fraction of lenders.

A marginal decrease in the equilibrium q will produce $f(q)$ fewer investors or more lenders, as a fraction of the mass of lenders:

$$h(q) = \frac{f(q)}{1 - F(q)} \tag{5.1a}$$

Expression (5.1a) is termed in other settings the "hazard rate". I would like to consider a distribution that parameterizes this hazard rate in a simple way while simultaneously being sufficiently flexible. One distribution that satisfies these criteria is the Weibull distribution, which is as follows:

$$F(z) = 1 - e^{-\left(\frac{z-1}{\lambda}\right)^{\omega}}, \lambda > 0, \omega > 0, \quad \mathcal{Z} \equiv [1, +\infty) \tag{5.1b}$$

For the Weibull distribution, the hazard rate is

$$h(q) = \frac{\omega}{\lambda}\left(\frac{q-1}{\lambda}\right)^{\omega-1} \tag{5.1c}$$

Then, because $q > 1$ in equilibrium, it is obvious that $h(q)$ is a decreasing function when $\omega < 1$, is constant when $\omega = 1$ and is increasing when $\omega > 1$.

Figure 1 depicts the Weibull density function for different values of the shape parameter ω. I have set $\lambda = 1$ throughout; it turns out that the value of this scale parameter does not alter the qualitative results. For $\omega < 1$, the shape of the density is similar to that of the Pareto distribution: a high mass of efficient agents is concentrated near $z = 1$. When $\omega = 1$, the distribution corresponds to the exponential distribution. For $\omega > 1$, the density becomes hump-shaped, with a large mass of relatively inefficient agents.

To compute the equilibrium price of equity, I use the following parametrization: $\alpha = 0.36, \beta = 0.95, \delta = 0.1$. These are standard values for a yearly macro model, and again, the qualitative results are insensitive to the actual values used. Equation (3.4a) cannot be solved analytically. I use Gauss-Legendre to compute the integral and a non-linear equation solver to find q. Figure 2 depicts the equilibrium q for different values of the financial depth parameter θ and for different values of ω,

$\omega = \{0.8, 0.9, 1, 1.2, 1.3, 1.4, 1.5\}$.

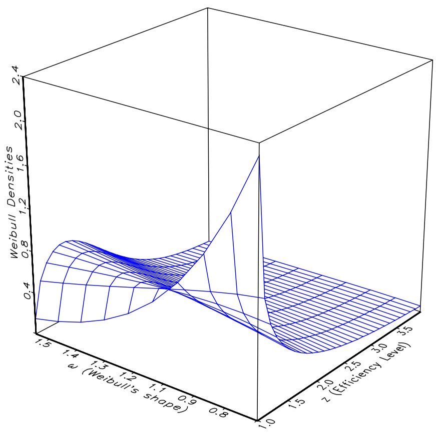

Figure 1: Weibull $F(z)$ for different ω

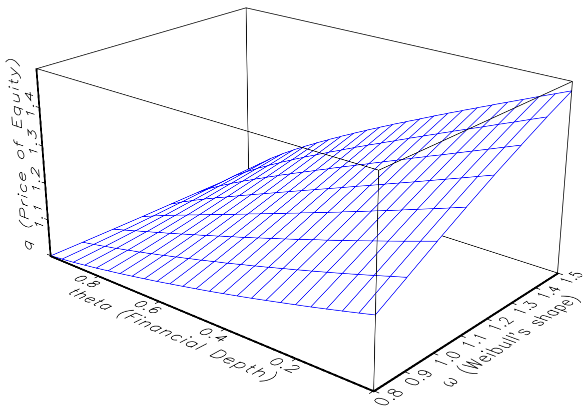

Figure 2: Equilibrium q for different θ, ω.

Figure 2 reveals several things. First, the equilibrium q is always larger than one. Second, q decreases with financial depth, as indicated by theory. Third, ω affects the equilibrium q; for a given θ, the equilibrium q is higher when ω is higher. This means that the fraction of investors for a given θ is larger the higher ω is. Note that financial deepening produces fewer investors in the economy. Because investors enjoy an advantage over lenders, it is likely that financial deepening produces greater inequality in this model, but as I explained above, I do not pursue issues of inequality in this paper.

It is easy to compute aggregate consumption as a fraction of capital using (4.3) in this economy, and the result is shown in Figure 3. This figure depicts consumption as a fraction of capital for different degrees of financial deepening and different values of ω. Figure 4 depicts the growth rate of the economy, computed with (3.4b). Several features of the figure deserve discussion. Note that for low levels of financial deepening, lower values of ω are associated with higher relative consumption and lower growth. This result is interesting under the following interpretation. Consider two dimensions of development, financial deepening and the efficiency with which the consumption good is transformed into capital. One could imagine that two countries with roughly the same level of financial development may differ in efficiency. Note that when $\omega < 1$, there is a larger mass of efficient agents, close to $z = 1$, than otherwise. This economy grows less than a less-efficient economy but is actually creating more capital and is able to finance a larger amount of consumption. Hence, less-developed economies grow more

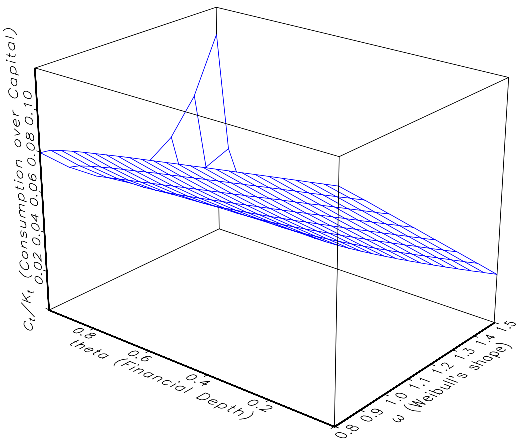

Figure 3: C_t/K_t for different ω, θ

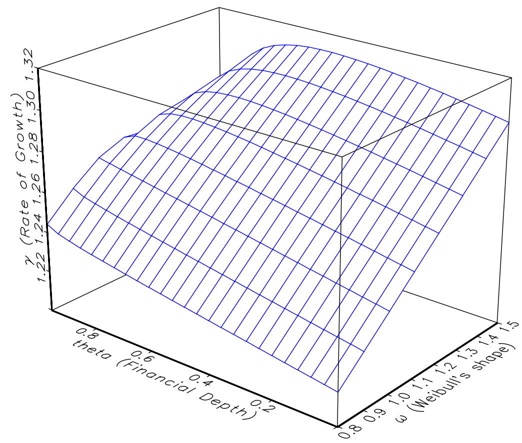

Figure 4: γ for different ω, θ

with lower consumption. This implication of the model accords well with empirical evidence. Let

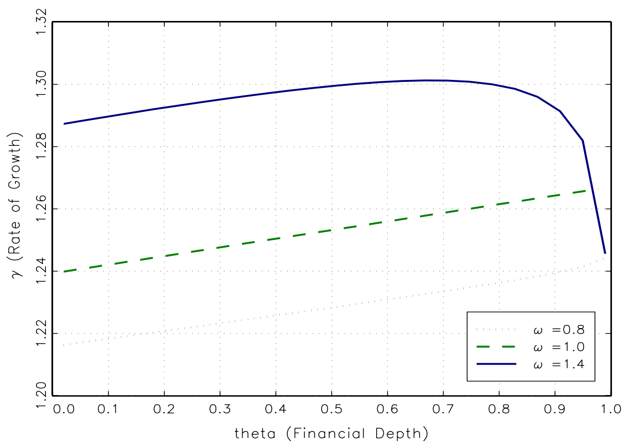

Figure 5: γ as a function of θ, for some values of ω.

us now focus on the effect of financial deepening. For $\omega \leq 1$, financial deepening decreases relative consumption and increases growth. As a partial and simple explanation, growth increases because credit flows more easily and finances capital creation, which translates into the overall productivity of the economy. Higher capital creation induces agents to consume a lower fraction of a larger amount of capital. This is only a partial explanation because it hinges on ω being less than one. In fact, for $\omega > 1$, a seemingly strange but interesting result emerges. With higher levels of financial depth, the initial positive influence on growth ceases; this can be seen in Figure 4. The effect of financial deepening on growth is also plotted in the plane (θ, γ) in Figure 5, to portray the relationship in a more transparent way. We can then see that growth actually decreases with financial deepening for parameterizations with $\omega > 1$, and we can see that consumption over capital increases when this occurs, as depicted in Figure 3.

Let me again refer to equation (3.14a). We know that increases in θ decrease q. We know that when $\omega > 1$, the "hazard rate" (5.1c) decreases when q decreases. Intuitively, $h(q)$ measures how inefficient the marginal agent is compared to the existing mass of inefficient agents. From the discussion in subsection 3.1, we know that an equilibrium requires that an increase in θ be accompanied by a larger mass of inefficient agents, so the expected return decreases to $1/\beta$. Because $h(q)$ decreases with a marginal decrease in q, the marginal mass of agents becomes more inefficient as the fraction of inefficient individuals decreases. This means that q needs to decrease faster to add a sufficient mass of lenders such that the additional net investment demand is satisfied while simultaneously reducing expected returns.[23] Recall equation (4.1), in which three effects have been identified on growth. Two of them, the *wealth* effect and the *extensive* margin effect, have a negative influence on growth, and the magnitude of this effect depends on how sensitive q is to an increase in θ. When the decline in q is larger, these effects are magnified. Moreover, the larger ξ is in absolute value, the less important the positive effect of the intensive margin is, and it may actually become negative.

5.1 Decomposition of the change in the growth rate

We can use equation (4.1) to decompose the change in the growth rate for different values of financial deepening. This decomposition provides a complete picture of the separate contributions of the three effects. Figure 6 depicts such a decomposition, which was performed for the case in which $\omega = 1.5 > 1$, namely, an increasing "hazard rate". In Figure 6, the change in the growth rate (the solid black curve) becomes negative for large values of θ. The growth rate is represented on the right axis to facilitate interpretation. We knew from theory that wealth effects and extensive margin effects are

[23]Note from Figure 2 that for $\omega > 1$, the equilibrium q decreases more steeply for large values of θ.

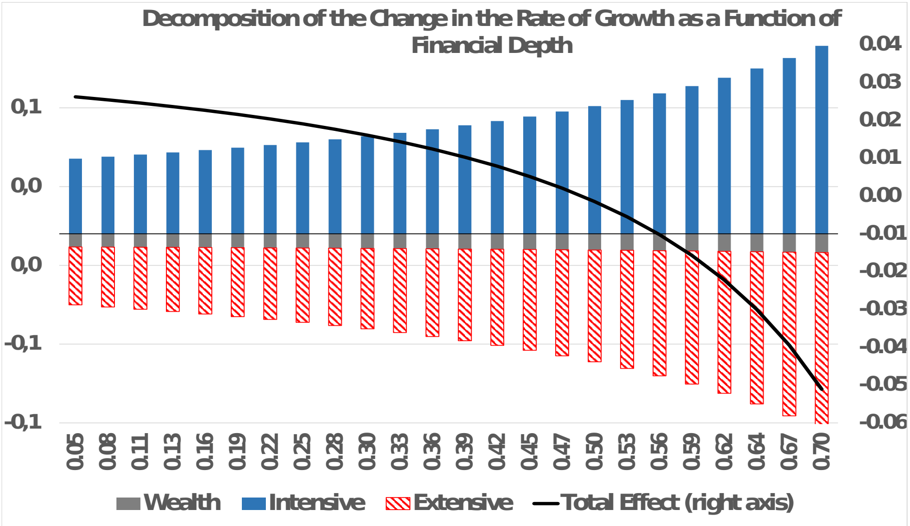

Figure 6: $d\gamma/d\theta$ as a function of θ

always negative. The figure reveals that the wealth effect is secondary to the extensive margin effect. The intensive margin effect is also important and always *positive.* Hence, it is never the case that the elasticity ξ is larger than one in absolute value. For larger values of θ, the negative effect of the extensive margin becomes more important, and while the positive effect of the intensive margin also increases, the overall change in the growth rate becomes negative.

6 Conclusions

This paper developed a stylized model of heterogenous agents and endogenous growth. The objective was to obtain a simple framework to investigate the extent to which financial deepening fosters growth and whether "excessive finance" harms growth. I developed this model based on a recent strand of papers starting with Kiyotaki and Moore (2005b) and Kiyotaki and Moore (2012) that include credit flows, in the form of equity, between agents who are able to invest and agents who are unable to invest. The key innovation of this paper is to endogenize the fraction of the population that decides to produce capital and hence becomes investors. While all individuals in the population could become investors, doing so is excessively costly for some agents. Agents facing a high cost of transforming the consumption good into capital will decide not to create capital and instead save in the form of purchases of claims on capital, thus becoming lenders.

For all individuals, the idiosyncratic costs of investment change over time. Current lenders equate the discount rate to the expected return on equity. This return is composed of the expected return conditional on becoming an investor and the expected return conditional on remaining a lender. Financial deepening positively affects this return. In equilibrium, to restore the equality between the discount rate and expected returns, the expected return conditional on remaining a lender must increase. By the law of large numbers, this means that the fraction of lenders must increase in a new equilibrium under financial deepening. It turns out that the shape of the distribution is important for determining whether this feature harms growth.

I used a Weibull distribution to model investment costs, which delivers a compact expression for the "hazard rate". If the hazard rate is increasing, then increases in financial deepening may decrease growth when the economy already exhibits high financial depth. It turns out that under an increase in financial deepening, restoring the equality between the discount rate and expected returns requires an increase in the fraction of lenders. Although this also means a smaller fraction of investors, it does not, per se, necessarily yield lower growth. The relevant factor is the relationship between the marginal mass of investors becoming lenders as a fraction of lenders. If this additional mass of lenders as a fraction of the initial mass of lenders decreases, then, eventually, the extra mass of lenders induced by further financial deepening is not sufficient to increase the fraction of lenders enough. In this case, the equity price must decrease steeply, which creates moderate negative wealth effects and strong negative extensive margin effects. The intensive margin effects are positive, but for high levels of financial depth, such effects do not outweigh the other two negative effects.

The results of this paper show that is possible to derive, from a simple stylized model, a result that has emerged in the empirical literature, namely the non-monotonic relationship between financial deepening and growth.

A The financial market structure

In this appendix, I explain the assumptions on the financial market structure that lead to equation (2.2). Imagine an entrepreneur at the end of the period determining the composition of her claims: $n_{t+1} = k_{t+1} - e_{t+1} + a_{t+1}$, where $e_{t+1} \geq 0$ is equity issued over capital and $a_{t+1} \geq 0$ is equity purchased issued by some other agent. Given that all claims issued are backed with capital and it is homogeneous, whenever $e_{t+1} > 0$, then $a_{t+1} = 0$. However, I allow for the possibility that they purchase claims on capital managed by someone else: $a_{t+1} > 0$ when $e_{t+1} = 0$. Thus, the balance sheet for any agent at the beginning of period t will take the form presented in Table 1.

assets:	liabilities:
$q_t k_t$	$q_t e_t$
$q_t a_t$	net worth:
	$q_t(k_t + a_t - e_t) = q_t n_t$

Table 1: Balance Sheet for an individual

There is a short-selling restriction on a_{t+1}; it is required to be positive:

$$a_{t+1} \geq 0. \tag{A.1}$$

Note that no "liquidity" constraint is imposed, as the individual can sell all previous holdings of equity issued by others. In addition to (A.1), there is another constraint on equity issued:

$$e_{t+1} \leq (1 - \delta)k_t + \theta x_t, \quad 0 < \theta < 1. \tag{A.2}$$

Restriction (A.2) states that equity can be issued up to all holdings of capital plus only up to a fraction θ of investment. Again, no "liquidity" constraint is imposed, as the agent is free to raise funds by issuing equity on the entire amount of existing capital.

Taking into account this financial structure, the feasibility set for any agent is given by

$$c_t + z_t x_t + q_t[a_{t+1} - (1-\delta)a_t] = w_t + n_t r_t + q_t[e_{t+1} - (1-\delta)e_t], \tag{A.3}$$

plus (A.1) and (A.2). Adding $q_t x_t$ to both sides of (A.3) and using (2.2a) and the definition of net worth, I obtain

$$c_t + z_t x_t + q_t[n_{t+1} - (1-\delta)n_t] = w_t + n_t r_t + q_t x_t, \tag{A.4}$$

which is (2.2b) in the text. By adding x_t to the negative of (A.2) and adding constraint (A.1), I obtain

$$n_{t+1} \geq (1-\theta)x_t, \tag{A.5}$$

which is constraint (2.2c).

Note that it does not matter how n_t is composed. Of course the *level* of claims and how it evolves for each individual will depend on the idiosyncratic uncertainty, but its composition is not needed to characterize the solution.

B Proofs

Lemma 1.

Proof. Using the guess in (3.5a), we see that the comparison between p_t and $\beta\mathbb{E}[\mathcal{D}_{t+1}(z')]$ for investors and q_t and $\beta\mathbb{E}[\mathcal{D}_{t+1}(z')]$ for lenders is key to their decisions to purchase equity and consume. In particular, $\beta\mathbb{E}[\mathcal{D}_{t+1}(z')]$ is the discounted expected marginal benefit from a unit of equity, which is compared to the cost of acquiring equity, and the latter differs among individuals. Five cases may arise.

i) $\beta\mathbb{E}[\mathcal{D}_{t+1}(z')] < p_t < q_t$, $p_t = \beta\mathbb{E}[\mathcal{D}_{t+1}(z')] < q_t$, iii) $p_t < \beta\mathbb{E}[\mathcal{D}_{t+1}(z')] < q_t$, iv) $p_t < q_t = \beta\mathbb{E}[\mathcal{D}_{t+1}(z')]$ and v) $p_t < q_t < \beta\mathbb{E}[\mathcal{D}_{t+1}(z')]$.[24]

Note that under $0 < \theta < 1$, cases i) through iii) cannot arise in equilibrium because no lender will be willing to purchase any claims. In case i), furthermore, entrepreneurs are not motivated to create any capital. Case v) can also be excluded because no agents would ever consume. The only possible equilibrium entails case iv).

To identify the undetermined coefficients in the value function, I assume that both investors and savers consume nothing.

Under such an assumption, the value function is

$$\mathcal{C}_t(z) + \mathcal{D}_t(z)n = \begin{cases} 0 + \beta\mathbb{E}\mathcal{C}_{t+1}(z') + \beta\mathbb{E}\mathcal{D}_{t+1}(z')\frac{w_t + [r_t + q_t(1-\delta)]n_t}{p_t} & \text{if } z \leq q_t \\ 0 + \beta\mathbb{E}\mathcal{C}_{t+1}(z') + \beta\mathbb{E}\mathcal{D}_{t+1}(z')\frac{w_t + [r_t + q_t(1-\delta)]n_t}{q_t} & \text{if } z > q_t. \end{cases} \tag{B.1}$$

Equating coefficients yields

$$\mathcal{C}_t(z) = \begin{cases} \beta\mathbb{E}\mathcal{C}_{t+1}(z') + \beta\mathbb{E}\mathcal{D}_{t+1}(z')\frac{w_t}{p_t} & \text{if } z \leq q_t \\ \beta\mathbb{E}\mathcal{C}_{t+1}(z') + \beta\mathbb{E}\mathcal{D}_{t+1}(z')\frac{w_t}{q_t} & \text{if } z > q \end{cases}, \quad \mathcal{D}_t(z) = \begin{cases} \beta\mathbb{E}\mathcal{D}_{t+1}(z')\frac{r_t + q_t(1-\delta)}{p_t} & \text{if } z \leq q_t \\ \beta\mathbb{E}\mathcal{D}_{t+1}(z')\frac{r_t + q_t(1-\delta)}{q_t} & \text{if } z > q_t \end{cases}. \tag{B.2}$$

Let me begin with $\mathcal{D}_t(z)$. Because under case iv) above $q_t = \beta\mathbb{E}\mathcal{D}_{t+1}(z')$,

$$\mathcal{D}_t(z) = \begin{cases} \frac{q_t}{p_t}[r_t + q_t(1-\delta)] & z \leq q_t \\ r_t + q_t(1-\delta) & z > q_t \end{cases} \tag{B.3}$$

Turning to $\mathcal{C}_t(z)$, taking expectations yields

$$\mathbb{E}\mathcal{C}_t(z) = \beta E\mathcal{C}_{t+1}(z') + w_t\left[\int_1^{q_t} \frac{q_t}{p_t} dF + 1 - F(q_t)\right] \tag{B.4}$$

Let me denote $\tilde{\mathcal{C}}_t(z) = \frac{\mathcal{C}_t(z)}{K_t}$. Dividing the equation above by K_t and focusing on a stationary environ-

[24] Note that I have already established that $p_t < q_t$, and hence, other cases are not considered.

ment in which q_t is constant, we have

$$\mathbb{E}\tilde{\mathcal{C}}(z) = \beta\gamma E\tilde{\mathcal{C}}(z) + A(1-\alpha)\left[\int_1^q \frac{q}{p}dF + 1 - F(q)\right] \tag{B.5}$$

Then,

$$\mathcal{C}_t(z) = \begin{cases} \frac{\gamma\beta A(1-\alpha)}{1-\beta\gamma}\left[\int_1^q \frac{q}{p}dF + 1 - F\right]K_t + \frac{q}{p}A(1-\alpha)K_t & z \leq q_t \\ \frac{\gamma\beta A(1-\alpha)}{1-\beta\gamma}\left[\int_1^q \frac{q}{p}dF + 1 - F\right]K_t + A(1-\alpha)K_t & z > q_t \end{cases} \tag{B.6}$$

Note that under the assumption of stationarity, $\mathcal{D}_t(z)$ does not depend on time. This completes the proof for the existence of value functions. The policy functions are directly derived under the equilibrium case iv). □

Proposition 2

Proof. I use the following form of equation (3.4a):

$$\Gamma_l(q) \equiv \frac{1}{\beta\mathcal{R}} = \int_1^q \frac{q}{p}dF(z) + [1 - F(q)] \equiv \Gamma_r(q). \tag{B.7}$$

Where $\mathcal{R}$ is defined in (3.14b). The following are properties of the functions defined in (B.7):

$$\Gamma_l(1) = 1, \qquad \Gamma_r(1) = 1 \tag{B.8a}$$

$$\Gamma_l(1/\theta) = \frac{1}{\theta + \beta(1-\delta)(1-\theta)}, \quad \Gamma_r(1/\theta) = \left[1 - F\left(\frac{1}{\theta}\right)\right] + \frac{1-\theta}{\theta}\int_1^{\frac{1}{\theta}} \frac{1}{z-1}dF(z) \tag{B.8b}$$

$$\frac{d\Gamma_l(q)}{dq} = \frac{r}{\beta[r + q(1-\delta)]^2} > 0, \quad \frac{d\Gamma_r(q)}{dq} = \int_1^q \frac{(1-\theta)z}{(z-q\theta)^2}dF(z) > 0 \tag{B.8c}$$

$$\frac{d^2\Gamma_l(q)}{dq^2} = -\frac{2r(1-\delta)}{\beta[r + q(1-\delta)]^3} < 0, \quad \frac{d^2\Gamma_r(q)}{dq^2} = \int_1^q \frac{(1-\theta)\theta z}{(z-q\theta)^3}dF(z) + \frac{f(q)}{(1-\theta)q} > 0 \tag{B.8d}$$

$$\lim_{q\to 1}\frac{d\Gamma_l(q)}{dq} = \beta r, \qquad \lim_{q\to 1}\frac{d\Gamma_r(q)}{dq} = 0. \tag{B.8e}$$

The relationships in (B.8) can be summarized in Figure 7. Both functions depart from $q = 1$ and $\Gamma_r > \Gamma_l$ at $q = 1/\theta$ under condition (3.11). As $q \to 1$, the slope of Γ_l is higher than the slope of

Γ_r independently of the parameter values. And Γ_l is strictly concave, while Γ_r strictly convex for all relevant values of q. This means that both functions cross only once.[25] The figure also shows the

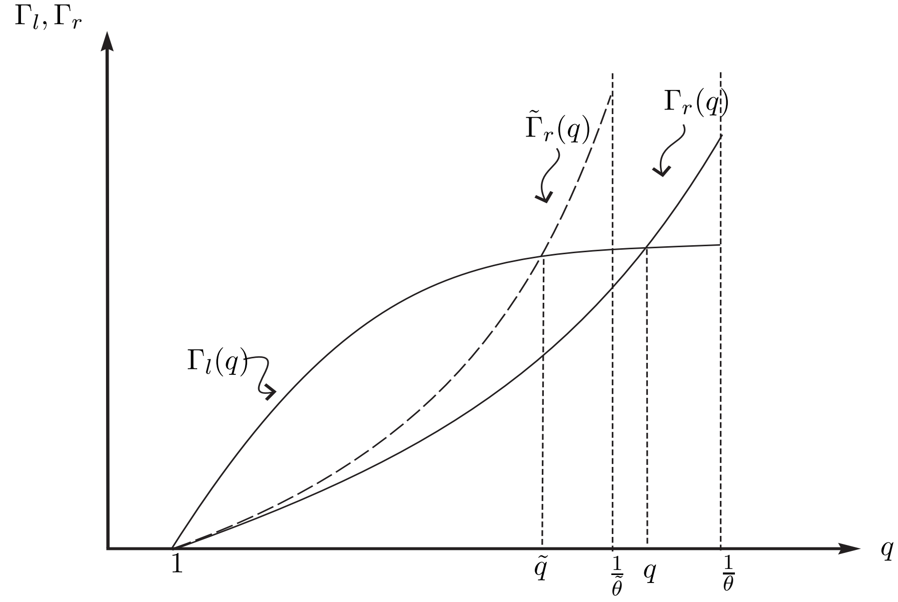

Figure 7: Existence. An economy with $\tilde{\theta} > \theta$, displays $\tilde{\Gamma}(q)$, and lower price $\tilde{q}$.

consequences of a higher θ. In this case, Γ_r rotates counterclockwise to $\tilde{\Gamma}_r$, and a new equilibrium $\tilde{q}$ is reached.

Note that if one considers a different value of A than that in (3.10), some results would differ. In particular, lower values will shift the curve Γ_r upward, but uniqueness will not be modified, as the two relevant curves will still cross once. Higher values of A will change the outcomes in more fundamental ways; please refer to footnote 19 for a brief discussion. □

Proposition 3

[25]Note that $q = 1$ cannot be an equilibrium for general $F(z)$. If $q = 1$, then there might be a measure zero of investors who are actually indifferent between whether to consume or invest, and thus, positive investment is not supported in equilibrium. This would be the case, for example, for the Weibull distribution with parameter $\omega > 1$, as analyzed in section 5. If we assume $F(z)$ to be the Dirac delta function at $z = 1$, then the model collapses to a representative agent model in which all individuals are indifferent between consuming and investing.

Proof. By the implicit function theorem in (B.7)

$$\frac{dq}{d\theta} = \frac{\frac{\partial \Gamma_r}{\partial \theta}}{\frac{\partial \Gamma_l}{\partial q} - \frac{\partial \Gamma_r}{\partial q}} \tag{B.9}$$

Note that

$$\frac{\partial \Gamma_r}{\partial \theta} = \int_1^q q \frac{q-z}{(z-\theta q)^2} dF(z) > 0 \tag{B.10}$$

Now, since both functions Γ_l, Γ_r are continuous, and because the equilibrium is unique, then near the equilibrium, the following inequality must hold:

$$\frac{\partial \Gamma_r}{dq} > \frac{\partial \Gamma_l}{dq} \tag{B.11}$$

and the result follows.[26] □

Proposition 4

Proof. This proof consists of several steps. First I show that the support of the stationary distribution is countable infinite. To this end, I consider the normalized policy functions for equity, expressed from (3.6a) and Assumption 1 as follows:

$$\tilde{n}_{t+1} \equiv \frac{g_{t+1}(\tilde{n}_t, z)}{K_{t+1}} = \begin{cases} \frac{A(1-\alpha)+[r+q(1-\delta)]\tilde{n}_t}{p\gamma}, & z \leq q \\ \zeta, & z > q \end{cases}, \tag{B.12}$$

where $\tilde{n}_t = n_t/K_t$ is normalized claims on capital. From any initial position in this normalized state space, individuals will eventually attain ζ as asset holdings and remain there when their efficiency draw is above q. When facing a draw $z \leq q$, they would use all resources to invest and accumulate claims. It

[26] Referring back to the discussion in footnote 19 in Proposition 2 about higher values of A. If two equilibria are found, the result in Proposition 3 is still valid for the high growth equilibrium. The high growth equilibrium was numerically found to be also the high asset price equilibrium, and then the slopes in (B.11) are satisfied near that equilibrium.

follows that all agents will hold equity only in the states defined by the following recursion:

$$\tilde{n}_{i+1} = \frac{A(1-\alpha) + [r + q(1-\delta)]\tilde{n}_i}{p\gamma}, \quad i = 1, 2, 3, \ldots, \tilde{n}_1 = \zeta. \tag{B.13}$$

This difference equation has a unique solution given by (4.4c). Note that it will not be possible to impose a policy function for becoming an inefficient agent in (B.12), whereby he will attain ζ from any level of equity accumulated previously, if a liquidity constraint were incorporated, as it will not be possible to sell equity over existing units of capital freely up to ζ.

Second, I demonstrate the special nature of the distribution of individuals by assets. Because there are two types of individuals in the economy at each point in time, the equation of motion in (2.5) resolves into two parts:

$$\Psi(\tilde{n}') = F(q)\Psi\left(\frac{p\gamma\tilde{n}' - A(1-\alpha)}{r + q(1-\delta)}\right) + 1 - F(q) \tag{B.14}$$

Note that because the relevant state has been normalized by the stock of capital, the economy is stationary, and hence, we look for a stationary measure of individuals. Equation (B.14) coupled with (B.13) implies the following:

$$\Psi(\tilde{n}_{i+1}) = F(q)\Psi\left(\frac{p\gamma\tilde{n}_{i+1} - A(1-\alpha)}{r + q(1-\delta)}\right) + 1 - F(q) = F(q)\Psi(\tilde{n}_i) + 1 - F(q), \tag{B.15}$$

which is a difference equation with boundary initial condition $\Psi(\tilde{n}_1) = 1 - F(q)$. The solution to this difference equation is

$$\Psi(\tilde{n}_i) = 1 - F(q)^i. \tag{B.16}$$

Note that the support of (B.16) is given by the recursion in (B.13), but this recursion has solution (4.4c), and therefore, the support of the distribution of individuals by assets can be defined over n_{it}, as was done in (4.4a), as $n_{it}/K_t = \tilde{n}_i$. □

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